DESIGN AND OPTIMIZATION OF A MULTICASCADE REFRIGERATION SYSTEM FOR USE IN EXTREME TEMPERATURE ENVIRONMENTS

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SIGNATURE PAGE

THESIS: DESIGN AND OPTIMIZATION OF A MULTICASCADE REFRIGERATION SYSTEM FOR USE IN EXTREME TEMPERATURE ENVIRONMENTS

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ABSTRACT

A novel refrigeration system concept is proposed to facilitate the cooling of electronics equipment operating in the atmosphere of Venus. The multi-cascade, multiple working fluid system is developed first as a thermodynamic framework which is then optimized such that system work is minimized. This optimization includes determination of optimal state point locations via a genetic algorithm as well as trade studies between refrigerants used in the loop cooling the electronics. To facilitate the high temperature heat rejection, exotic working fluids with high critical points are investigated for use in the rejection loop. The resulting optimized cycle resulted in a net operating power requirement of 101.5 W compared to an unoptimized design of 127 W, a power savings of 20 %.
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CHAPTER 1: Introduction

Surface exploration of the planet Venus has been a challenging mission since landers have first been sent to the planet for investigation. The environmental conditions on Venus are harsh including a highly dense atmosphere, surface atmospheric pressure 92 times that of Earth, surface temperature of 462°C with diurnal variance of 0.5°C, sulfuric acid precipitation, and a rocky, volcanic, desert surface (NASA, 2016). These conditions have made surface exploration missions short-lived and problematic. The Soviet space program Venera was the first program to attempt exploration of Venus with the Venera 3 being the first probe to make it to the surface. Many of the early Venera space probes failed upon entry to the planet’s atmosphere due to the high pressures which were unexpected. Later successful probes in the Venera program typically averaged around 63 minutes of operation before succumbing to thermal, battery or range failure. The NASA Pioneer Multiprobe, although not a lander, survived 45 minutes following surface impact.

There is recent renewed interest, to create a lander to explore the Venetian surface for more lengthy periods of time. Survival times in the Venera program had improved overtime by using a refrigeration system to pre-cool the electronics prior to entry as well as using insulating phase change materials to allow prolonged operation time before failure due to thermal runaway (NASA, 2016). This option does not offer continuous operation and can only extend life before inevitable thermal failure. Another option is the use of extreme temperature electronics but this option increases costs significantly, uses exotic materials and the elevated temperatures naturally lead to degraded performance of the electronics. A desired solution would be a cooling system which could continuously
operate and support more standard electronics equipment. This is a challenging system to develop for a multitude of reasons including large difference in environment temperatures, remote power management, and high heat rejection temperature.

The proposed thermodynamic cycle for this system is a multiple working fluid, multi-cascade refrigeration cycle operating between the temperature limits of the environment and required electronics payload temperature. As will be shown, this cycle development will consist of multiple degrees of freedom which will make cycle optimization difficult. The objective is to minimize the work required thereby maximizing the coefficient of performance of the system. As the design space shape is unknown and in higher order of dimension, it lends itself well to use of a genetic algorithm to optimize the problem.
The basic framework of the thermodynamic cycle must first be developed in a parameterized way such that the state points which serve as the system’s degrees of freedom may be changed and constrained to optimize the cycle. The most basic refrigeration cycle is the reversed Rankine cycle and consists ideally of the following processes:

a) Isentropic compression
b) Constant pressure condensation
c) Isenthalpic throttling
d) Constant pressure evaporation

Whereby heat is absorbed from the cooled region during the evaporation process and is rejected to the environment during the condensation process. In the ideal case listed above

Figure 1. Ideal Vapor Compression Refrigeration Cycle (Cengel, 2011)
no irreversible losses are associated with the system and this may be regarded as the Carnot refrigeration cycle, providing the maximum theoretical performance of the system. Realistically, the compression process is likely to heat the working fluid due to friction in the compressor and pressure losses will occur through the condenser, evaporator, and throttling devices. To compensate for irreversibilities in the compressor, it is common to modify the compression process by including the effect of an isentropic efficiency. While pressure losses throughout the other system components will affect the compressor, and cause the system to work harder, it is assumed the compressor efficiency will play the largest role in the total required work input. Other deviations from the idealized cycle are necessary to realize a practical system design, such as ensuring a superheated state prior to the compressor inlet to avoid two phase flow, but such deviations are not considered in this analysis except where noted.

For many industrial applications cascading systems are used to increase efficiency of the overall system. Cascading systems tie different refrigeration cycles together by combining the evaporator of one cycle and the condenser of the other cycle into a single heat exchanger. In this manner, the heated vapor condensing from one cycle is vaporizing the condensed liquid in the other. This construction of cycles can be visualized in Figure 2.
The complexity introduced into the design of the system by cascading cycles is offset by the increased efficiency they can provide. Additionally, it is sometimes necessary to utilize multiple working fluids or achieve larger temperature differences both of which are the case in the presented design.

A unique problem becomes evident as the cycle is constructed, primarily with concern to heat rejection to the environment. To properly reject heat to the environment, the environment temperature will need to be several degrees less than that of the condenser temperature. This ensures that a significant enough temperature difference is present to allow for heat rejection to the environment. This concept is illustrated in the T-s diagram of Figure 3.
Considering the environment temperature of Venus, this would require a working fluid capable of condensing at a temperature above 462°C and therefore must have critical point above this temperature. Fluids with such high critical temperatures are exceedingly rare. A survey of available fluids conducted by the author has shown that only two primary fluids exist with this property: sulfuric acid ($\text{H}_2\text{SO}_4$) at 655°C (Muller, 2016) and a group of substances deemed fatty acid methyl esters (FAME) with critical points between 485°C and 526°C (Lemmon, 2013). Initial investigation suggests that sulfuric acid will be extremely challenging to use as a working fluid due to its corrosive nature. Additionally, sulfuric acid has a challenging phase diagram depending on acid content in the aqueous solution utilized including various gas phases being present that would hamper heat exchanger performance and pose aeration issues. Finally, sulfuric acid in any solution content does not have widely available thermophysical properties currently and would first need characterization before its feasibility could be studied. For these reasons, a FAME substance was chosen to be the working fluid in the heat rejection cycle. Specifically, methyl linoleate ($\text{C}_{19}\text{H}_{34}\text{O}_2$ or MLL) was chosen as it has a critical temperature of 526°C,
offering the highest critical point of the FAME substances and ample head room above the Venetian surface temperature to work with. Additionally, thermophysical properties are available through use of either the National Institute of Standards and Technology’s (NIST) REFPROP library or the freely available CoolProp. Despite this, these factors primarily encourage the analysis to proceed, however it should be understood that characterization of methyl linoleate will be necessary to determine feasibility as a working fluid. No such applications of any FAME substance are known to the author and typical applications of these fluids has been as biodiesel fuel and detergents (Anneken, 2006). A saturation curve for methyl linoleate as output from REFPROP are shown in Figure 4.

Figure 4. Saturation Curve p-H Diagram for Methyl Linoleate

The lower cycle which removes heat from the electronics package directly will be a standard fluid refrigerant cycle as described by Figure 1. Part of this cycle’s optimization
will be a trade study of the refrigerant to be used. In between this standard heat absorption cycle and the FAME heat rejection cycle, are a set of transcritical and supercritical carbon dioxide cycles that will be cascaded as well. The transcritical cycle will be connected to the lower standard cycle and transfer heat to the supercritical carbon dioxide cycle, which in turn will transfer heat to the methyl linoleate cycle. The transcritical cycle operates CO$_2$ as the working fluid within the saturation dome for the evaporation process but crosses the saturation curve to reject heat above the critical point in a sensible heat process. As the name implies, the supercritical cycle takes place completely in the supercritical region of fluid properties. Figure 5 illustrates an example of a transcritical process for R-744.

![Figure 5. Example of a Transcritical Process for R-744 (Staub, 2004)](image)

These CO$_2$ processes add more degrees of freedom to the overall cycle but are hypothesized as necessary to achieve the temperature rise to reject heat to the environment. Whereas
many other cycle points are constrained by Rankine cycle relations and saturation curve properties, the presence of sensible heat exchange in these cycles leads to unconstrained temperatures in the exit states of the cooling process of the cycle. These will need to be fixed for analysis and may be optimized against but will likely ultimately be functions of the physical system design such as heat exchanger packaging constraints.

With the overall cycle framework outlined, the relationships for the cycle may begin to be established. For illustrative purposes, a proposed initial design cycle is shown in Figures 6, 7, and 8. This standard set of charts and state point indexing will now be used as a reference for the development and established relationships.

Figure 6. P-h Diagram for Bottom Cycle
Figure 7. P-h Diagram for CO₂ Middling Cycles

Figure 8. P-h Diagram for MLL Topping Cycle
From the diagrams, it is found that the total cycle consists of a total of 16 state points. For each of these state points, it will be necessary to fully define them using two independent extensive properties. For labeling of these properties the following variables will be used:

\( h \) - specific enthalpy  
\( s \) – specific entropy  
\( v \) – specific volume  
\( T \) – temperature  
\( P \) – pressure  
\( X \) – vapor quality  
\( f \) – property at saturated liquid state (subscript)  
\( g \) – property at saturated vapor state (subscript)

Beginning with the bottoming cycle in Figure 6, this cycle is a simple ideal Rankine cycle with compressor isentropic efficiency compensation. State points 2 and 4 are immediately fixed once evaporator and condenser pressures are chosen as they both are in saturated states. This can be shown as:

\[
P_2 = P_{E1} \\
X_2 = 1 \\
P_4 = P_{HX1} \\
X_4 = 0
\]
Where \( P_{E1} \) is the evaporator pressure and \( P_{HX1} \) is the heat exchanger pressure between the bottoming cycle and lower middling cycle. With state point 4 fixed, state point 1 may be found using the fact that process 1 to 4 is isenthalpic:

\[
h_1 = h_4
\]
\[
P_1 = P_{E1}
\]

State point 3 is determined by correcting an isentropic process between state points 2 and 3 for the isentropic efficiency of the pump. This is expressed as:

\[
s_{3s} = s_2
\]
\[
P_3 = P_{HX1}
\]
\[
h_3 = h_2 + \frac{h_{3s} - h_2}{\eta_C}
\]

Where \( s_{3s} \) is the specific entropy under isentropic conditions, \( h_{3s} \) is the specific enthalpy under isentropic conditions and \( \eta_C \) is the compressor isentropic efficiency. Here it is noted that \( h_{3s} \) is determined from knowledge of \( P_3 \) and \( s_{3s} \). This set of relations fully defines the bottoming cycle and other state point information can now be extracted.

Additionally, with the heat transfer requirement from the electronics space known, required mass flows and power requirements may be determined. Now examining the carbon dioxide cycles, it is observed initially that only state point 6 is fixed as it is saturated:

\[
P_6 = P_{HX1}
\]
\[
X_6 = 1
\]

State point 8 lies in the super critical region and therefore is not a constrained point. This point must be specified in the design and will be controlled. Therefore, its state may be expressed as:
Where \( P_{HX2} \) is the heat exchanger pressure between the lower middling and upper middling cycle and \( T_{C1} \) is the specified exit temperature of the lower fluid stream from the heat exchanger. State point 5 is now determined noting process 8 to 5 is isenthalpic:

\[
h_5 = h_8
\]

\[
P_5 = P_{HX1}
\]

State point 7 may now be determined in the same fashion as state point 3 was determined in the bottoming cycle via correction of the isentropic process:

\[
s_{7s} = s_6
\]

\[
P_7 = P_{HX2}
\]

\[
h_7 = h_6 + \frac{h_{7s} - h_6}{\eta_C}
\]

Thus, analysis of the lower middling cycle is like that of the bottoming cycle except for an unconstrained exit temperature. The upper middling cycle is similar in analysis nature; however, two points now lie unconstrained by the saturation curve, thus two controlled temperatures are required. State point 10 is an unconstrained point and may be expressed as:

\[
P_{10} = P_{HX2}
\]

\[
T_{10} = T_{C2}
\]

Where \( T_{C2} \) is the specified exit temperature of the upper fluid stream from the heat exchanger. State point 12 is the other unconstrained point and is analogous to state point 8. This state is expressed as:

\[
P_{12} = P_{gc}
\]
\[ T_{12} = T_{c3} \]

Where \( P_{gc} \) is the pressure of the CO\(_2\) on the gas side of the upper heat exchanger and \( T_{c3} \) is the specified exit temperature of the upper CO\(_2\) stream from the upper heat exchanger.

With process 12 to 9 being isenthalpic, state point 9 becomes:

\[ h_9 = h_{12} \]
\[ P_9 = P_{HX2} \]

State point 11 may now be determined by again correcting an isentropic process between states 10 and 11:

\[ s_{11s} = s_{10} \]
\[ P_{11} = P_{gc} \]
\[ h_{11} = h_{10} + \frac{h_{11s} - h_{10}}{\eta_C} \]

This set of relations fully constrains the middling cycles and allows for determination of intermediate mass flows and compressor power requirements. Finally, examining the MLL cycle, it is nearly the same analysis as that of the standard cycle analysis used with the bottoming cycle. However, examining the saturation curve for MLL shows some challenges. Geometrically, the saturation dome is a thin region which is angled towards the direction of increasing enthalpy (see Figure 4). Since the entropy correction through the compressor is small, the compressor work process will appear as a nearly isenthalpic process. When this is coupled with the peculiar shape of the saturation curve it becomes necessary to consider superheating of the MLL out of the evaporator so that it does not re-condense as it becomes pressurized in the compressor. This effect can be observed in the compression process between points 14 and 15 shown in Figure 8. However, it is
desired to minimize this to avoid excessive heating. With this requirement, state points 15 and 16 will be fixed at the saturated states:

\[ P_{15} = P_C \]
\[ X_{15} = 1 \]
\[ P_{16} = P_C \]
\[ X_{16} = 0 \]

Where \( P_C \) is the condenser pressure. Although point 16 is consistent with the standard cycle analysis, fixing point 15 as a saturated vapor at the compressor exit ensures that no two-phase flow will occur in the compressor and is the minimum requirement for the needed superheat at state point 14. State point 13 is determined from the isenthalpic relationship:

\[ P_{13} = P_{E2} \]
\[ h_{13} = h_{16} \]

Where \( P_{E2} \) is the evaporator side pressure of the upper heat exchanger. State point 14 can be calculated by correcting an isentropic process between state point 14 and 15 which yields:

\[ s_{14s} = s_{15} \]
\[ P_{14} = P_{E2} \]
\[ h_{14} = h_{15} + \frac{h_{14s} - h_{15}}{\eta_C} \]

With these final relationships established, the entire cycle is now formed on a per unit mass basis. Given a required amount of heat removal from the electronics package, the fluid stream mass flows may be determined as well as the work requirement for the cycle.
Reviewing the cycle development, it has been shown that there are nine main variables which must be chosen by the designer to fix the entire cycle while the cycle relations give all other remaining information on a per unit mass basis. The two remaining practical figures which are needed is the heat removal requirement from the electronics package as well as the assumed compressor efficiency. The heat removal will control each cycle’s mass flow and serves to scale the system with demand. The compressor efficiency will control the compression process and is primarily a function of the compressor design. Of the nine parameters, the three CO₂ exit temperatures are least controllable. These will be directly controlled by the design of the heat exchangers themselves. These parameters will be fixed throughout the optimization process. Additionally, power consumption from the carbon dioxide cycles are small and will not be considered in the optimization. Furthermore, efficiency and heat removal will not be studied in the optimization as they directly affect the result and simply act as scaling variables for the cycle. Thus, the optimization problem is reduced from a 9-parameter problem to a 4-parameter study. Table 1 summarizes these key parameters and their initial choices prior to the cycle optimization. Table 2 provides a summary of the state points associated with the parameter choices in Table 1. These results are initially generated using R410A as the refrigerant in the electronics cooling loop however this will be examined during the optimization as well.
### Table 1
Cycle Control Parameters and Inputs

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description of parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{E1} )</td>
<td>0.45 MPa</td>
<td>Evaporator pressure for electronics refrigerant loop</td>
</tr>
<tr>
<td>( P_{E2} )</td>
<td>0.40 MPa</td>
<td>Evaporator (upper heat exchanger) pressure for MLL loop</td>
</tr>
<tr>
<td>( P_{HX1} )</td>
<td>2 MPa</td>
<td>Refrigerant to ( CO_2 ) heat exchanger pressure</td>
</tr>
<tr>
<td>( P_{HX2} )</td>
<td>8 MPa</td>
<td>( CO_2 ) to ( CO_2 ) heat exchanger pressure</td>
</tr>
<tr>
<td>( P_{GC} )</td>
<td>12 MPa</td>
<td>Gas cooler (upper heat exchanger) pressure for ( CO_2 ) loop</td>
</tr>
<tr>
<td>( P_C )</td>
<td>0.7 MPa</td>
<td>Condenser pressure (MLL)</td>
</tr>
<tr>
<td>( T_{C1} )</td>
<td>20 °C</td>
<td>Exit temperature of ( CO_2 ) to ( CO_2 ) heat exchanger for transcritical loop</td>
</tr>
<tr>
<td>( T_{C2} )</td>
<td>60 °C</td>
<td>Exit temperature of ( CO_2 ) to ( CO_2 ) heat exchanger for supercritical loop</td>
</tr>
<tr>
<td>( T_{C3} )</td>
<td>30 °C</td>
<td>Exit temperature of ( CO_2 ) from gas cooler (upper heat exchanger)</td>
</tr>
<tr>
<td>( \eta_C )</td>
<td>80 %</td>
<td>Assumed isentropic efficiency for all cycle compressors</td>
</tr>
<tr>
<td>( Q )</td>
<td>100 W</td>
<td>Heat removal required from electronics package</td>
</tr>
</tbody>
</table>
Table 2

State Points for Unoptimized Cycle

<table>
<thead>
<tr>
<th>State Point</th>
<th>T (°C)</th>
<th>P (MPa)</th>
<th>υ (m³/kg)</th>
<th>h (kJ/kg)</th>
<th>s (kJ/kg/K)</th>
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<tr>
<td>1</td>
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<td>0.0191</td>
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<td>3</td>
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<td>1.876</td>
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<td>4</td>
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<td>2.00</td>
<td>0.0010</td>
<td>252.3</td>
<td>1.177</td>
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<td>5</td>
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<td>8</td>
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<td>9</td>
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<td>458.1</td>
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<td>11</td>
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<td>0.0040</td>
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<tr>
<td>12</td>
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<td>12.0</td>
<td>0.0012</td>
<td>265.6</td>
<td>1.194</td>
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<tr>
<td>13</td>
<td>467.7</td>
<td>0.40</td>
<td>0.0326</td>
<td>1467</td>
<td>3.605</td>
</tr>
<tr>
<td>14</td>
<td>467.7</td>
<td>0.40</td>
<td>0.0421</td>
<td>1579</td>
<td>3.760</td>
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<tr>
<td>15</td>
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<td>0.70</td>
<td>0.0193</td>
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<tr>
<td>16</td>
<td>476.6</td>
<td>0.70</td>
<td>0.0020</td>
<td>1467</td>
<td>3.600</td>
</tr>
</tbody>
</table>

The cycle work and coefficient of performance were determined assuming a 100 W load from the electronics enclosure and are found to be 127 W and 0.787 respectively.
CHAPTER 3: Cycle Optimization

Utilizing the framework established in Chapter 2, it is desired to optimize the cycle as to minimize the work requirement. To accomplish this the relations developed for the cycle were coded into a solving function in MATLAB. The function accepts the 9 cycle control parameters indicated in Table 1 and evaluates all state points, mass flows, and energy transfers through the cycle. In addition to this, a MATLAB toolbox has been constructed around the function’s output structure to generate the thermodynamic diagrams and other useful data. The solver function is necessary to not only solve the cycle but to iterate upon using some sort of optimization method until a minimization of cycle work occurs.

Optimization procedures rely on performing detailed parametric analysis of a problem which produces an objective or fitness function. An optimization engine then operates on this objective function until this objective function is optimized, often with respect to certain design constraints. The objective function operates on a vector of parameters and produces potentially a vector of outputs. The solver function in the case of the cycle optimization plays the role of the objective function as it outputs the system work. The eleven cycle control parameters are the vector of parameters to operate on and the state points, mass flows and energy transfers comprise the vector of output parameters.

The goal of most optimization routines is to seek the global optimizer which is the vector of input parameters which optimizes the objective function. The shape of the objective function becomes important as it can cause difficulty to arise in determining the global optimizer. For example, discontinuities and local minima can both make searching for a global optimizer difficult (Sudhoff, 2014). This difficulty stems from gradient based
methods such as Newton’s method which rely on the derivative to exist to operate successfully. For functions containing discontinuities this will cause gradient based methods to fail as the derivative is not defined at the discontinuity. Additionally, these methods can have difficulty in determining the global optimizer in as it will instead determine a local optimizer. Routine convergence is determined by driving the gradient of the objective function to zero. However, this method is sensitive to the initial solution estimate. If the initial estimate is located near a local optimizer the routine will tend to converge to the local optimizer since these routines will tend to optimize in a small neighborhood about the initial estimate (Sudhoff, 2014). To circumvent these issues, a population based algorithm can be used which will search multiple initial solution estimates to avoid becoming trapped in local optimizer or fail at a discontinuity. Figure 8 gives an example of the difficulties which may be encountered.

Figure 9. Discontinuities and Local Minima in Fitness Functions (Sudhoff, 2014)

A particularly powerful population based method is the genetic algorithm which employs principles of biology such as mutation, crossover, and selection to recreate the natural
selection process. This process often takes longer to converge on a similar solution as more iterations or generations are required to ensure adequate selection but often produces stable solutions heuristically. While the parameter space for the optimization of the cycle is four dimensional and not easily visualized, it is not expected to contain many if any discontinuities due to the smooth nature of the thermophysical properties as produced by NIST REFPROP. Additionally, these thermophysical properties are typically monotonically varying functions thus local minima are not expected as well. However, the genetic algorithm is chosen to provide a stable base optimization routine that will allow for greater ease in scaling the optimization problem to include additional state points that could possibly introduce failures in gradient based solvers. Additionally, population based routines are easily extendable to multi-objective optimization, another future optimization goal.

Tools for genetic optimization exist for use with MATLAB including Purdue’s Genetic Optimization System Engineering Toolbox (GOSET) as well as MATLAB’s own product within its Optimization Toolbox. MATLAB’s Optimization Toolbox was ultimately chosen as it additionally provides a built in graphical tool to ease problem setup. Additionally, the GUI produces live plots tracking problem convergence across generations as well as other useful data. Figure 10 shows the graphical interface utilized by MATLAB’s Optimization Toolbox.
The optimization problem must be stated and constrained mathematically first before the optimization procedure can occur. The optimization problem can be broken into two parts: optimization of the refrigerant cycle and optimization of the MLL cycle. Thus, the two cycles can be optimized independently and results combined to yield the fully optimized cycle. For both cycles this then becomes a problem of determining the proper pressure levels which minimize the work for each cycle. This is easiest done by determining the lower cycle pressure and the pressure ratio. As the figure of interest is the total cycle work, this will be the fitness function. Therefore, the optimization problem is stated as:
\[ W = f(\mathbf{x}) \]
\[ f(\mathbf{x}^*) \leq W \]
\[ \mathbf{x} = \begin{bmatrix} P_L \\ \alpha \end{bmatrix} \]

Where \( W \) is the cycle total compressor work, \( f \) is the fitness function, \( \mathbf{x} \) is the parameter vector upon which \( f \) operates, \( P_L \) is the lower pressure of the cycle, \( \alpha \) is the cycle pressure ratio and \( \mathbf{x}^* \) is the parameter vector which minimizes the fitness function. This statement applies to both the refrigerant and MLL cycle. Constraints for each cycle however are different to ensure proper operation. The lower pressure of the refrigerant cycle must be such that the saturation temperature of the fluid used lies below the maximum allowed temperature of the electronics payload. Likewise, the upper pressure of the MLL cycle must ensure the saturation temperature through the condenser is above the Venetian surface temperature. Finally, the constraint of the coefficient of performance not exceeding Carnot must apply. These constraints may be given as:

\[ T_{L,R} < T_{EL} \]
\[ T_{H,MLL} > T_{env} \]
\[ \beta < \beta_{\text{Carnot}} \]

Where \( T_{L,R} \) is the evaporator temperature for the refrigerant loop, \( T_{EL} \) is the allowable electronics temperature, \( T_{H,MLL} \) is the condenser temperature for the MLL loop, \( T_{env} \) is the Venus environment temperature, \( \beta \) is the coefficient of performance and \( \beta_{\text{Carnot}} \) is the Carnot or maximum theoretical coefficient of performance.

Before the optimization algorithm is used to analyze the cycle, a refrigerant trade study is to be done. Two candidate refrigerants were proposed: R-410A and NH\(_3\). To study the effect the fluid has on the cycle the control parameters listed in Table 1 were used to
first construct the cycle. Then sweeping the same parameters intended for use in the optimization, $P_L$ and $\alpha$, the COP of the overall cycle was determined. Sweeping these parameters in the refrigerant cycle indicate the varying of the cycle control parameters $P_{E1}$ and $P_{HX2}$. The results of the parameter sweep are presented in Figure 11.

![COP vs Pressure Ratio](image)

Figure 11. COP vs Pressure Ratio at Various Pressure Levels for R-410A and NH₃

Inspection of the chart indicates that in all scenarios, NH₃ consistently outperforms R-410 as a refrigerant by resulting in a higher coefficient of performance. Therefore, NH₃ is chosen to be the refrigerant to be used in the bottoming cycle.

First, optimization of the bottoming cycle was performed with NH₃ as the refrigerant. Using the constraints set forth previously, the search space was set to sweep
through pressures between 105 kPa and 405 kPa as well as pressure ratios of 5 to 12. The terminal convergence plot to the average value of the cycle work across generations is shown in Figure 12.

![Figure 12. Generation Convergence Plot of Bottoming Cycle Optimization](image)

The optimization results show that the first generation resulted in a power requirement of 149.4 W which after 72 generations was reduced to 119.3 W, a 20 % reduction in total compressor work across the optimization. This was achieved at an evaporator pressure of 405 kPa and pressure ratio of 7.45. Additionally, a check on the constraints show that the evaporator temperature for this case is -1.55 °C and cycle COP of 0.71. This is compared to the desired maximum electronics temperature of 100 °C and Carnot COP of 1.02.

Next, optimization of the topping cycle was performed. During this optimization, the optimal parameters for the bottoming cycle were used in the inputs. Using the constraints set forth previously, the search space was to set a sweep through pressures
between 605 kPa and 665 kPa as well as pressure ratios of 1.05 to 2. The small ranges given to the sweep space is due to the minimum temperature requirement of 462°C. Thus the range is small between this pressure and the critical pressure. The terminal convergence plot to the average value of the cycle work across generations is shown in Figure 13.

![Figure 13. Generation Convergence Plot of Topping Cycle Optimization](image)

The optimization results show that the first generation resulted in a power requirement of 116.8 W which after 51 generations was reduced to 101.5 W a 13 % reduction in total compressor work across the optimization. This was achieved at the evaporator pressure of 605 kPa and a pressure ratio of 1.05. Additionally, a check on the constraints show that the condenser temperature for this case is 469.5 °C and a cycle COP of 0.98. This is compared to the surface temperature of 462 °C and Carnot COP of 1.022. These results represent the fully optimized cycle with the final Stobridge number of 96 % being achieved, which is the ratio of actual COP to Carnot COP. Table 3 lists the results of the optimized cycle while Tables 4 and 5 detail the cycle mass flows and required compressor powers. The figures following graphically show these results in thermodynamic diagram format.
<table>
<thead>
<tr>
<th>State Point</th>
<th>T (°C)</th>
<th>P (MPa)</th>
<th>υ (m³/kg)</th>
<th>h (kJ/kg)</th>
<th>s (kJ/kg/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.553</td>
<td>0.405</td>
<td>0.0809</td>
<td>523.4</td>
<td>2.191</td>
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<tr>
<td>2</td>
<td>-1.553</td>
<td>0.405</td>
<td>0.3058</td>
<td>1460.6</td>
<td>5.642</td>
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<tr>
<td>3</td>
<td>183.5</td>
<td>3.02</td>
<td>0.0690</td>
<td>1851.6</td>
<td>5.818</td>
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<tr>
<td>4</td>
<td>65.97</td>
<td>3.02</td>
<td>0.0019</td>
<td>523.4</td>
<td>2.044</td>
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<tr>
<td>5</td>
<td>-5.344</td>
<td>3.02</td>
<td>0.0037</td>
<td>246.9</td>
<td>1.178</td>
</tr>
<tr>
<td>6</td>
<td>-5.344</td>
<td>3.02</td>
<td>0.0121</td>
<td>433.5</td>
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<tr>
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<td>1.903</td>
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<td>8.00</td>
<td>0.0012</td>
<td>246.9</td>
<td>1.148</td>
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<tr>
<td>9</td>
<td>25.69</td>
<td>8.00</td>
<td>0.0013</td>
<td>265.6</td>
<td>1.211</td>
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<tr>
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<td>60.00</td>
<td>8.00</td>
<td>0.0052</td>
<td>458.1</td>
<td>1.829</td>
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<td>0.0040</td>
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<td>30.00</td>
<td>12.0</td>
<td>0.0012</td>
<td>265.6</td>
<td>1.194</td>
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<tr>
<td>13</td>
<td>468.6</td>
<td>0.605</td>
<td>0.0039</td>
<td>1443</td>
<td>3.568</td>
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<tr>
<td>14</td>
<td>468.6</td>
<td>0.605</td>
<td>0.0237</td>
<td>1571</td>
<td>3.741</td>
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<tr>
<td>15</td>
<td>469.5</td>
<td>0.635</td>
<td>0.0219</td>
<td>1572</td>
<td>3.742</td>
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<tr>
<td>16</td>
<td>469.5</td>
<td>0.635</td>
<td>0.0020</td>
<td>1443</td>
<td>3.568</td>
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</table>
Table 4

Mass Flow Rates and Compressor Power for Each Fluid Loop

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Mass Flow Rate (kg/hr)</th>
<th>Compressor Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH₃</td>
<td>0.384</td>
<td>41.7</td>
</tr>
<tr>
<td>TCO₂</td>
<td>2.73</td>
<td>37.7</td>
</tr>
<tr>
<td>SCO₂</td>
<td>3.35</td>
<td>20.8</td>
</tr>
<tr>
<td>MLL</td>
<td>5.63</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Table 5

Overall Cycle Summary

<table>
<thead>
<tr>
<th>Cycle Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Power</td>
<td>101.6 W</td>
</tr>
<tr>
<td>COP</td>
<td>0.984</td>
</tr>
<tr>
<td>Stobridge Number</td>
<td>0.96</td>
</tr>
</tbody>
</table>


Figure 14. T-s Diagram, Optimized Bottoming Cycle

Figure 15. P-h Diagram, Optimized Bottoming Cycle
Figure 16. P-v Diagram, Optimized Bottoming Cycle

Figure 17. T-s Diagram, Optimized Middling Cycles
Figure 18. P-h Diagram, Optimized Middling Cycles

Figure 19. P-v Diagram, Optimized Middling Cycles
Figure 20. T-s Diagram, Optimized Topping Cycle

Figure 21. P-h Diagram Optimized Topping Cycle
Figure 22. P-v Diagram, Optimized Topping Cycle
CHAPTER 4: Practical Considerations

Throughout the design and optimization of the cycle in the previous chapters, there have been several considerations given to practical design of the thermodynamic system. For example, ensuring the presence of superheat to avoid two-phase compressor flow in the MLL cycle. However, there do exist other issues which must be addressed to realize a fully functioning system.

The most prominent issue challenging the system practicality is the use of methyl linoleate as a working fluid in the topping cycle. The major reported application of methyl linoleate or any FAME for that matter has been as a biodiesel fuel. Two risks that this may pose are both in terms of flammability and longevity. Considering that FAMEs are derived from biological substances such as rapeseed oil and vegetable oil, extensive life testing will need to be conducted to ensure that the working fluid will not break down under extended usage or to gain understanding on what type of lifecycle the fluid does have before losing function. The major reason for which FAME was chosen as a candidate as a working fluid was due to the high critical point allowing heat exchange with the hot Venetian atmosphere and availability of thermophysical properties for use in analysis. Thermophysical properties for MLL were found to be available in two software, CoolProp and REFPROP. Both software work in essentially the same manner in determining the fluid properties required. Ultimately REFPROP was chosen as it is developed by a nationally recognized lab as a standard for various chemical properties. NIST officially designates REFPROP as a product of its Standard Reference Data and has the official title as “NIST Reference Fluid Thermodynamic and Transport Properties Database”. This naming convention often leads
to confusion and misunderstanding that REFPROP is a program and not a database. This is key to note because REFPROP contains no actual experimental information except for critical and triple points for pure fluids. Rather the program utilizes equations for thermodynamic and transport properties to calculate state points that a user requests in regards to a fluid or mixture. REFPROP uses equations of state (EOS) to predict fluid properties using the most accurately known parameters and applicable EOS.

Historical EOS that have been used for predicting fluid are typically not robust enough to provide accurate property predictions over a large range of temperatures and pressures for pure fluids and mixtures. For example, the Benedict-Webb-Rubin and modified Benedict-Webb-Rubin offers good data fitting by utilizing 8 parameter variables to fluid data that provides good accuracy for densities less than 2.5 times that of the critical density. However, one of the most popular multi-parameter EOS are those based off the Helmholtz energy as an explicit function of non-dimensional, or reduced temperature and density which are given by:

\[ a(\delta, \tau) = a^*(\delta, \tau) + a^r(\delta, \tau) \]

\[ \delta = \frac{\rho}{\rho_c} \]

\[ \tau = \frac{T_c}{T} \]

Where \( a \) is the Helmholtz function, \( a^* \) is the ideal gas contribution, \( a^r \) is the residual portion, \( \delta \) is the reduced density, \( \tau \) is the reduced temperature, \( \rho_c \) is the critical density, \( T_c \) is the critical temperature, \( \rho \) is the density, and \( T \) is the temperature. The ideal gas contribution
can be determined via thermodynamic relations however, the residual term must be fitted to available data and typically has the form:

\[ a^r = \sum N_k \delta^{lk} \tau^{jk} + \sum N_k \delta^{lk} \tau^{jk} e^{-\delta^{lk}} \]

Which can be extended to include an arbitrary number of terms to ensure a good fit to measured data. Typically, each summation term in the residual term consists of 4-20 terms (Sonntag, 2013) which indicates 24 to 120 parameters typically used. This type of fluid characterization scheme is utilized internally to REFPROP and each fluid’s associated parameters are pulled from verified and referenced papers. Utilizing REFPROP’s fluid information and examining the article detailing the development of the FAME model, the authors note that limited measured data was performed and/or available. Although the paper conclusion shows close agreement with the measured data, the limited sampling could lead to incorrectly predicted properties, especially near the critical point where many models require more sampling or give large error. This further exemplifies the need for sweeping characterization of the fluid to ensure it can perform the intended job as much of the design relies on this being feasible.

Additionally, the results have shown that MLL exists mostly in the cycle simply for its ability to reject heat during a phase change in such an extreme environment. With this being the case, it calls into question another needed study on the utilization of the middling cycles. One possibility would be to eliminate the CO\(_2\) cycles all together and couple NH\(_3\) and MLL with a simply cascading system. This eliminates some of the complexity of the system design as CO\(_2\) is difficult to seal typically requiring expensive, heavy mechanical seals. The attractive fluid properties of CO\(_2\) could possibly make it easier to pump and lower
losses in the system if it were kept, especially in a remote environment where power is at a premium. This however calls in to question the need for two CO₂ cycles. It may also be possible to benefit from the CO₂ fluid properties while only maintaining one of the cycles, likely the supercritical loop. This is another option that should be explored.

Finally, the issue of powering the system will be a challenge. Although the system is highly optimized the given environment only permits a maximum theoretical coefficient of performance of nearly one. Therefore, at the minimum every unit of heat energy removed requires a unit of electrical power. Some preliminary options are being investigated including use of a radioisotope thermal electric generator (RTG) or promising new advancements in lithium carbon dioxide (LiCO₂) based fuel cells and batteries (Xu, 2013). The LiCO₂ option is highly attractive considering that Venus’s atmosphere is 96.5% CO₂ (NASA, 2016) and could possibly one of the first interplanetary vehicles to consume fuel on location, however this technology is in its infancy and will likely need advanced development and testing before ready for space applications.
CHAPTER 5: Conclusion & Future Work

The development of multiple working fluid, multi-cascading refrigeration system was presented. The system was successfully constrained and optimized in parts through use of a genetic optimization algorithm inside of MATLAB’s Optimization Toolbox resulting in a power consumption reduction of 20% from the unoptimized state. This resulted in an optimized total power consumption of 101.5 W. Additionally, a trade study was performed on the utilization of R410A or NH$_3$ as the refrigerant operating in the electronics cooling loop. It was determined that across multiple pressure levels and multiple pressure ratios that NH$_3$ consistently resulted in a higher COP and was thus chosen to be the working fluid for that loop. Concerns with the use of methyl linoleate as a refrigerant were discussed including its unknown longevity and potentially unproven thermophysical properties. Finally, concerns with powering the system in a remote environment were discussed with early options identified as either RTG’s or LiCO$_2$ based fuel cells.

Looking towards the future of this design, the optimization results showed the relatively unimportant role that methyl linoleate plays in the cycle. Aside from achieving the necessary rejection temperatures most work is done by the NH$_3$ and CO$_2$ cycles. Considering this conclusion, it would be ideal to reduce the system complexity by considering direct coupling of the bottoming and topping cycles by eliminating the CO$_2$ cycles. Thus, further optimization is desired by including the middling cycles in the optimization analysis. Furthermore, it is also desired to determine other design metrics necessary to design against such as weight, strength and cost such that a multi-objective design optimization can be performed.
Current work under way for this design includes characterization of methyl linoleate’s thermophysical properties and life cycle testing as a refrigerant. This characterization project is currently funded and in the build phase. Additionally, design of the compressor needed to handle methyl linoleate is underway with expected preliminary results to be completed by late December. Finally, if trade studies show that the CO₂ cycles do serve benefit to the design, porous media heat exchanger sizing studies have been conducted and are ready to use for hardware selection.
REFERENCES


Anneken, David J.; Both, Sabine; Christoph, Ralf; Fieg, Georg; Steinberner, Udo; Westfechtel, Alfred (2006). "Fatty Acids". *Ullmann's Encyclopedia of Industrial Chemistry*

function [C] = solver(I,bottomfluid)
% solver
%=====================================================================
% FUNCTION: solver.m
%
% DESCRIPTION: A function which calculates all parameters of the
% refrigeration cycle.
%
% INPUTS: I - a vector of input data
% (1) - Pressure of evaporator #1 [kPa]
% (2) - Pressure of evaporator #2 [kPa]
% (3) - Pressure of heat exchanger #1 [kPa]
% (4) - Pressure of heat exchanger #2 [kPa]
% (5) - Pressure of gas cooler [kPa]
% (6) - Pressure of condenser [kPa]
% (7) - Cooler #1 exit temperature [K]
% (8) - Cooler #2 exit temperature [K]
% (9) - Cooler #3 exit temperature [K]
% (10) - Isentropic efficiency of compressors
%
% OUTPUTS: C - a data structure containing all the cycle analysis data
% .T - a structure containing all calculated temperature state points
% .x - temperature of state point 'x' [K]
% .P - a structure containing all calculated pressure state points
% .x - pressure of state point 'x' [kPa]
% .h - a structure containing all calculated enthalpy state points
% .x - enthalpy of state point 'x' [J/kg]
% .s - a structure containing all calculated entropy state points
% .x - entropy of state point 'x' [J/kg/K]
% .v - a structure containing all calculated specific volume state points
% [m^3/kg]
% .W - a structure containing all calculated work values
% .W1 - work done by compressor #1
% .W2 - work done by compressor #2
% .W3 - work done by compressor #3
% .W4 - work done by compressor #4
% .W - total work done by compressors

% .Q - a structure containing all heat transferred values
% .Q1 - heat rejected from low cycle to low mid
% .Q2 - heat rejected from low mid to hi mid
% .Q3 - heat rejected from hi mid to top
% .QL - heat removed from space
% .QH - heat rejected to atmosphere

% INTERNAL:
% P_evap1 - Pressure of evaporator #1 [kPa]
% P_evap2 - Pressure of evaporator #2 [kPa]
% P_HX1 - Pressure of heat exchanger #1 [kPa]
% P_HX2 - Pressure of heat exchanger #2 [kPa]
% P_gascooler - Pressure of gas cooler [kPa]
% P_condenser - Pressure of condenser [kPa]
% T_coolerexit1 - Cooler #1 exit temperature [K]
% T_coolerexit2 - Cooler #2 exit temperature [K]
% T_coolerexit3 - Cooler #3 exit temperature [K]
% eta_s - Isentropic efficiency of compressors
% Q_lift - Required heat removal by system [W]

% AUTHOR: Chris McNamara, EIT
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% Mechanical Engineering Department
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% Pomona, CA 91764
% chris.m.mcnamara@gmail.com
% cmmcnamara@cpp.edu

% DATE WRITTEN: 01/15/2016

% MODIFICATION HISTORY:
% 01/15/2016 - Added introductory comments.

% NOTES:
%
%======================================================================

%Extraction of data from input vector
%==
P_evap1=I(1);
P_evap2=I(2);
P_HX1=I(3);  
P_HX2=I(4);  
P_gascooler=I(5);  
P_condenser=I(6);  
T_coolerexit1=I(7);  
T_coolerexit2=I(8);  
T_coolerexit3=I(9);  
eta_s=I(10);  
Q_lift=I(11);  

%Fluids of interest are specified here  
%=====================================  
TopFluid='MLINOLEA';  
MidFluid='CO2';  
BotFluid=bottomfluid;  

%%%%%Bottom Cycle Analysis%%%%  
 ----  
%standard refrigeration
%Evaporator exit/Compressor inlet enthalpy
h2=refpropm2D('H','P',P_evap1,'Q',ones(1,size(P_evap1,2)),BotFluid);  
%Condenser exit/valve inlet enthalpy
h4=refpropm2D('H','P',P_HX1,'Q',zeros(1,size(P_evap1,2)),BotFluid);  
%Valve exit/evaporator inlet enthalpy
h1=h4;  
%Bottom cycle mass flow
m_bot=Q_lift./(h2-h1);  
%Evaporator outlet entropy
s2=refpropm2D('S','P',P_evap1,'Q',1,BotFluid);  
%Isentropic compressor outlet entropy
s3s=s2;  
%Isentropic compressor outlet enthalpy
h3s=refpropm2D('H','P',P_HX1,'S',s3s,BotFluid);  
%Redefine h2 to matrix to match h3s (single vs dual parameter dependence)
h2=h2*ones(1,size(h3s,1));  
%Actual compressor outlet enthalpy
h3=h2+((h3s-h2).'/eta_s);  
%Compressor work
W_c_bot=m_bot.*(h3-h2);  
%Heat rejected to upper cycle
Q_H_bot=m_bot.*(h3-h4);  
%Remaining state points
T2=refpropm2D('T','P',P_evap1,'Q',1,BotFluid);  
P2=P_evap1;  
r2=refpropm2D('D','P',P_evap1,'Q',1,BotFluid);  
v2=r2.^-1;  
P4=P_HX1;  
T4=refpropm2D('T','H',h4,'P',P4,BotFluid);  
s4=refpropm2D('S','P',P4,'H',h4,BotFluid);  
r4=refpropm2D('D','P',P4,'H',h4,BotFluid);  
v4=r4.^-1;  
P3=P4;  
T3=refpropm2D('T','P',P3,'H',h3,BotFluid);
s3 = refpropm2D('S', 'T', T3, 'H', h3, BotFluid);

r3 = refpropm2D('D', 'P', P3, 'H', h3, BotFluid);

v3 = r3.^-1;

P1 = P2;
T1 = T2;

s1 = refpropm2D('S', 'T', T1, 'H', h1, BotFluid);

r1 = refpropm2D('D', 'P', P1, 'H', h1, BotFluid);

v1 = r1.^-1;

%%%%%%% Low-Mid Cycle Analysis %%%%%%%

% Evaporator exit/Compressor inlet enthalpy
h6 = refpropm2D('H', 'P', P_HX1, 'Q', 1, MidFluid);

% Gas cooler exit/Valve inlet temperature
T8 = T_coolerexit1;

h8 = refpropm2D('H', 'T', T8, 'P', P_HX2, MidFluid);

% Valve exit/evaporator inlet enthalpy
h5 = h8;

% Mid-low cycle mass flow
m_midl = m_bot.*(h3 - h4)/(h6 - h5);

% Evaporator outlet entropy
s6 = refpropm2D('S', 'P', P_HX1, 'Q', 1, MidFluid);

% Isentropic compressor outlet entropy
s7s = s6;

% Isentropic compressor outlet enthalpy
h7s = refpropm2D('H', 'P', P_HX2, 'S', s7s, MidFluid);

% Actual compressor outlet enthalpy
h7 = h6 + ((h7s - h6) ./ eta_s);

% Compressor work
W_c_midl = m_midl.*(h7 - h6);

% Heat rejected to upper cycle
Q_H_midl = m_midl.*(h7 - h8);

% Remaining state points
T6 = refpropm2D('T', 'P', P_HX1, 'Q', 1, MidFluid);
P6 = P_HX1;

r6 = refpropm2D('D', 'P', P_HX1, 'Q', 1, MidFluid);

v6 = r6.^-1;

P7 = P_HX2;

T7 = refpropm2D('T', 'P', P7, 'H', h7, MidFluid);

s7 = refpropm2D('S', 'T', T7, 'P', P7, MidFluid);

r7 = refpropm2D('D', 'T', T7, 'P', P7, MidFluid);

v7 = r7.^-1;

P8 = P_HX2;

s8 = refpropm2D('S', 'T', T8, 'P', P8, MidFluid);

r8 = refpropm2D('D', 'T', T8, 'P', P7, MidFluid);

v8 = r8.^-1;

P5 = P_HX1;

T5 = T6;

s5 = refpropm2D('S', 'P', P5, 'H', h5, MidFluid);

r5 = refpropm2D('D', 'P', P5, 'H', h5, MidFluid);

v5 = r5.^-1;

%%%%%%% High-Mid Cycle Analysis %%%%%%%

% Lower gas cooler exit/Compressor inlet pressure
\[ P_{10} = P_{\text{HX2}}; \]
% Lower gas cooler exit/Compressor inlet temperature
\[ T_{10} = T_{\text{coolerexit2}}; \]
% Lower gas cooler exit/Compressor inlet enthalpy
\[ h_{10} = \text{refpropm2D('H', 'T', T_{10}, 'P', P_{10}, \text{MidFluid})}; \]
% Lower gas cooler exit/Compressor inlet entropy
\[ s_{10} = \text{refpropm2D('S', 'T', T_{10}, 'P', P_{10}, \text{MidFluid})}; \]
% Isentropic compressor outlet entropy
\[ s_{11s} = s_{10}; \]
% Upper gas cooler pressure
\[ P_{11} = P_{\text{gascooler}}; \]
% Isentropic compressor outlet enthalpy
\[ h_{11s} = \text{refpropm2D('H', 'P', P_{11}, 'S', s_{11s}, \text{MidFluid})}; \]
% Actual compressor outlet enthalpy
\[ h_{11} = h_{10} + \left( \frac{h_{11s} - h_{10}}{\eta_s} \right); \]
% Upper gas cooler exit temperature
\[ T_{12} = T_{\text{coolerexit3}}; \]
% Upper gas cooler pressure
\[ P_{12} = P_{\text{gascooler}}; \]
% Upper gas cooler exit enthalpy
\[ h_{12} = \text{refpropm2D('H', 'T', T_{12}, 'P', P_{12}, \text{MidFluid})}; \]
% Valve exit enthalpy
\[ h_{9} = h_{12}; \]
% Cycle mass flow
\[ m_{\text{midh}} = m_{\text{midl}} \times \frac{(h_{7} - h_{8})}{(h_{10} - h_{9})}; \]
% Compressor work
\[ W_{c_{\text{midh}}} = m_{\text{midh}} \times (h_{11} - h_{10}); \]
% Heat rejected to upper cycle
\[ Q_{H_{\text{midh}}} = m_{\text{midh}} \times (h_{11} - h_{12}); \]
% Remaining state points
\[ P_{9} = P_{10}; \]
\[ s_{9} = \text{refpropm2D('S', 'P', P_{9}, 'H', h_{9}, \text{MidFluid})}; \]
\[ T_{9} = \text{refpropm2D('T', 'P', P_{9}, 'H', h_{9}, \text{MidFluid})}; \]
\[ r_{9} = \text{refpropm2D('D', 'P', P_{9}, 'H', h_{9}, \text{MidFluid})}; \]
\[ v_{9} = r_{9}^{-1}; \]
\[ r_{10} = \text{refpropm2D('D', 'P', P_{10}, 'H', h_{10}, \text{MidFluid})}; \]
\[ v_{10} = r_{10}^{-1}; \]
\[ T_{11} = \text{refpropm2D('T', 'P', P_{11}, 'H', h_{11}, \text{MidFluid})}; \]
\[ s_{11} = \text{refpropm2D('S', 'P', P_{11}, 'H', h_{11}, \text{MidFluid})}; \]
\[ r_{11} = \text{refpropm2D('D', 'P', P_{11}, 'H', h_{11}, \text{MidFluid})}; \]
\[ v_{11} = r_{11}^{-1}; \]
\[ s_{12} = \text{refpropm2D('S', 'P', P_{12}, 'H', h_{12}, \text{MidFluid})}; \]
\[ r_{12} = \text{refpropm2D('D', 'P', P_{12}, 'H', h_{12}, \text{MidFluid})}; \]
\[ v_{12} = r_{12}^{-1}; \]
% Compressor outlet state
\[ P_{15} = P_{\text{condenser}}; \]
\[ h_{15} = \text{refpropm2D('H', 'P', P_{15}, 'Q', 1, \text{TopFluid})}; \]
\[ s_{15} = \text{refpropm2D('S', 'P', P_{15}, 'H', h_{15}, \text{TopFluid})}; \]
% Condenser exit state
\[ P_{16} = P_{\text{condenser}}; \]
\[ h_{16} = \text{refpropm2D('H', 'P', P_{16}, 'Q', 0, \text{TopFluid})}; \]
% Valve exit/evaporator state
P13=P_evap2;
h13=h16;
\%Evaporator exit/compressor inlet state
P14=P_evap2;
s14s=s15;
h14s=refpropm2D('H', 'P', P14, 'S', s14s, TopFluid);
\%Mass flows
m_top=m_midh.*(h11- \ h12)./(h14- \ h13);
\%Compressor work
W_c_top=m_top.*(h15- \ h14);
\%Heat rejected to upper cycle
Q_H_top=m_top.*(h15- \ h16);
P14=P_evap2;
r14=refpropm2D('D', 'P', P14, 'H', h14, TopFluid)';
v14=r14. ^ -1;
P16=P_condenser;
T16=refpropm2D('T', 'H', h16, 'P', P16, TopFluid);
s16=refpropm2D('S', 'P', P16, 'H', h16, TopFluid);
r16=refpropm2D('D', 'P', P16, 'H', h16, TopFluid)';
v16=r16. ^ -1;
P15=P16;
T15=refpropm2D('T', 'P', P15, 'H', h15, TopFluid);
s15=refpropm2D('S', 'T', T15, 'H', h15, TopFluid);
r15=refpropm2D('D', 'P', P15, 'H', h15, TopFluid)';
v15=r15. ^ -1;
P13=P14;
T13=T14;
s13=refpropm2D('S', 'P', P13, 'H', h13, TopFluid);
r13=refpropm2D('D', 'P', P13, 'H', h13, TopFluid)';
v13=r13. ^ -1;
\%Overall cycle calculations
\%==================================
W_c_total=W_c_bot+W_c_midl+W_c_midh+W_c_top;
Q_total=Q_lift+W_c_total;
\%Sto=COP/COP_Carnot;
\%Redefinition of outputs from previous code labels
\%===============================================
COP=Q_lift ./W_c_total;